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EPFL

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Name : .....

Signature :

Midterm Exam
MATH-232 Probability and Statistics
Instructor: Prof. Emmanuel Abbe
Spring 2023
Duration : 20.04.2023 from 13:15 to 14:30.

Do not turn the page before the start of the exam. This document is double-sided and has 12 pages. Do not unstaple.

- First and foremost, write your name on this paper and also write your SCIPER number using the checkbox provided above.
- Place your student card on your table.
- **No other paper materials** are allowed to be used during the exam.
- Using a **calculator** or any electronic device is **not permitted** during the exam.
- A cheat sheet is provided on the last pages of this booklet.
- For the **multiple choice** questions, you will receive:
  +3 points if your answer is correct,
  0 points if you give no answer or more than one,
  −1 points if your answer is incorrect.
- Use a **black or dark blue ballpen** and clearly erase with **correction fluid** if necessary.
- The multiple choice questions are shuffled, and hence are not in the order of difficulty.
- If a question is wrong, the teacher may decide to nullify it.

Respectez les consignes suivantes | Observe this guidelines | Beachten Sie bitte die unten stehenden Richtlinien		
choisir une réponse | select an answer Antwort auswählen	ne PAS choisir une réponse | NOT select an answer NICHT Antwort auswählen	Corriger une réponse | Correct an answer Antwort korrigieren
☒ ☑ ■	☐	☐

ce qu'il ne faut **PAS** faire | what should **NOT** be done | was man **NICHT** tun sollte

First part: multiple choice questions

For each question, mark the box corresponding to the correct answer. Each question has **exactly one** correct answer. No justifications are needed for this part.

Question 1 There are 3 different permutations of the word BOB (that give different words): BOB, OBB, and BBO. Which of the following words has the largest number of different permutations?

- ☐ HAMID
- ☐ ARIANA
- ☐ APOLLO
- ☐ HANNAH

Question 2 Let $X \sim \text{Uniform}(1,3)$ and Y such that $Y|X \sim \exp(X)$, i.e., $f_{Y|X}(y|x) = xe^{-yx}$ for $y \in (0,\infty), x \in (1,3)$. What is the value of $\mathbb{E}[X^2Y]$?

- ☐ 4
- ☐ 2
- ☐ 1
- ☐ 9

Question 3 Assume that in a society people's weights follow a normal distribution with mean $\mu = 71$ and variance $\sigma^2 = 9$. The percentage of people with weights less than 65 is closest to

- ☐ 2.28%
- ☐ 15.87%
- ☐ 25.2%
- ☐ 0.13%

Question 4 Assume that $\theta \sim \mathcal{N}(0,1)$ (standard Normal distribution). Consider $X = \sin(\theta)$ and $Y = \cos(\theta)$. Which of the following statements is correct?

- ☐ X and Y are uncorrelated and dependent
- ☐ X and Y are correlated and dependent
- ☐ X and Y are correlated and independent
- ☐ X and Y are uncorrelated and independent

Question 5 A man tells the truth only 75% of times (in other words, he lies with probability 0.25). He has a biased coin that gives heads also with probability 0.75. In an experiment, the man flips his coin and says that the result is heads; what is the probability that the coin was really heads?

- ☐ $\frac{9}{16}$
- ☐ $\frac{10}{16}$
- ☐ $\frac{9}{10}$
- ☐ $\frac{3}{4}$

Question 6 Let X_n have Binomial distribution $X_n \sim \text{Bin}(n,p)$ such that $np = \lambda$ is constant. As $n \to \infty$, the probability $\mathbb{P}(X_n = 0)$ tends to

- ☐ $e^{-\lambda} - e^{-2\lambda}$
- ☐ $\lambda e^{-\lambda}$
- ☐ $e^{-\lambda}$
- ☐ $1 - e^{-\lambda}$

Question 7 Assume that $\mathbb{P}(C) = 0.5$, $\mathbb{P}(D) = 0.3$, and $\mathbb{P}(C|D) = 0.5$. What is the value of $\mathbb{P}(D^c|C^c)$? (C^c and D^c represent the complements of C and D respectively.)

- ☐ 0.7
- ☐ 0.4
- ☐ 0.6
- ☐ 0.8

Question 8 For random variables X and Y, we have $\mathrm{Var}(X) = 2$, $\mathrm{Var}(Y) = 8$ and $\mathrm{corr}(X, Y) = -0.5$. What is $\mathrm{cov}(2X + Y, Y - 3X)$?

- ☐ -2
- ☐ 2
- ☐ 0
- ☐ 4
- ☐ -4

Question 9 The random variable X has the density function

$$f_X(x) = \begin{cases} cx & \text{if } x \in [0, 2] \\ 0 & \text{otherwise} \end{cases},$$

where c is a constant which makes f_X a valid density. What is $\mathrm{Var}(X)$?

- ☐ $\frac{4}{3}$
- ☐ $\frac{1}{18}$
- ☐ $\frac{3}{4}$
- ☐ $\frac{2}{9}$

Cheat sheet

Basic formulas

- Properties of binomial coefficients
  (a) Pascal's triangle $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$.
  (b) Vandermonde's formula $\sum_{j=0}^{r} \binom{m}{j}\binom{n}{r-j} = \binom{m+n}{r}$.
  (c) Negative binomial series $(1-x)^{-n} = \sum_{i=0}^{\infty} \binom{n+i-1}{i} x^i, |x| < 1$.
  (d) $\lim_{n\to\infty} n^{-r}\binom{n}{r} = \frac{1}{r!}$, where $r \in \mathbb{N}$ is fixed.
- Inclusion-exclusion formula:

$$\mathbb{P}(\bigcup_{i=1}^{n} A_i) = \sum_{r=1}^{n}(-1)^{r+1} \sum_{1\le i_1<i_2<\ldots<i_r\le n} \mathbb{P}(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_r})$$

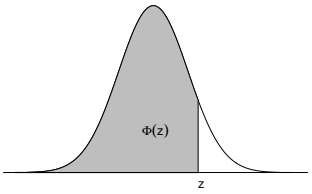
- For random variables X and $Y = g(X)$, where g is a monotone increasing or decreasing function with differentiable inverse g^{-1}, we have

$$f_Y(y) = |\frac{dg^{-1}(y)}{dy}| f_X(g^{-1}(y)).$$

Distributions

Distribution	PMF/PDF	Expected Value	Variance
Bernoulli Bern(p)	$\mathbb{P}(X=1)=p$ $\mathbb{P}(X=0)=1-p$	p	$p(1-p)$
Binomial Bin(n,p)	$\mathbb{P}(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$ $k\in\{0,1,2,\dots n\}$	np	$np(1-p)$
Geometric Geom(p)	$\mathbb{P}(X=k)=(1-p)^{k-1}p$ $k\in\{1,2,\dots\}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Negative Binomial NegBin(r,p)	$\mathbb{P}(X=x)=\binom{x-1}{r-1}p^r(1-p)^{x-r}$ $x\in\{r,r+1,r+2,\dots\}$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$
Hypergeometric HyperGeom(w,b,n)	$\mathbb{P}(X=k)=\frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}$ $k\in\{0,1,2,\dots,n\}$	$\mu=\frac{nw}{b+w}$	$\left(\frac{w+b-n}{w+b-1}\right)n\frac{\mu}{n}(1-\frac{\mu}{n})$
Poisson Pois(λ)	$\mathbb{P}(X=k)=\frac{e^{-\lambda}\lambda^k}{k!}$ $k\in\{0,1,2,\dots\}$	λ	λ
Uniform U(a,b)	$f(x)=\frac{1}{b-a}$ $x\in[a,b]$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential exp(λ)	$f(x)=\lambda e^{-\lambda x}$ $x\in(0,\infty)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Normal $\mathcal{N}(\mu,\sigma^2)$	$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ $x\in(-\infty,\infty)$	μ	σ^2
Gamma Gamma(α,λ)	$f(x)=\frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\lambda x}$ $x\in(0,\infty)$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$

Standard normal distribition $\Phi(z)$



For $z < 0$ we use symmetry: $\mathbb{P}(Z \leq z) = \Phi(z) = 1 - \Phi(-z)$, $z \in \mathbb{R}$.

z	0	1	2	3	4	5	6	7	8	9
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56750	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84850	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92786	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997